A Note on the Optimal Selection and Weighting of Comparable Properties

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***Abstract*. This paper reexamines the recommendation by Vandell (1991), Gau, Lai and Wang (1992, 1994) and Green (1994) for the use of the minimum variance and coefficient of variation criteria in the optimum selection of comparables. These authors under-emphasize the typical valuation scenario that involves extremely small samples. The analyst must rank the few available comparable properties and select the "best" to carry the most weight in the final estimate of value. Rank transformation regression is suggested as one approach that can be used to extract the buying trend. The commonly taught paired-sale analysis will remain as the industry tool until more accurate estimates of value are developed with small samples.**

Introduction

An interesting series of recent articles have offered several common statistical techniques for the selection of comparable properties to be used in the appraisal of a subject parcel. Vandell (1991) suggested that a minimum variance estimator is best for selecting and weighting comparable properties. Gau et al. (1992) presented the minimum coefficient of variation as the selection criterion, and Green (1994) presented a technique for determining which is the best approach. Gau et al. (1994) suggested that one approach cannot be judged as the best unless empirical evidence is available. The objective of this analysis is to find a market-derived unit of comparison that is reliable.

These arguments have missed two critical points that are derived from the classical, age-old discussion between Paul Samuelson and Milton Friedman on the premise that acceptable theory must be relevant and testable. The statistical techniques presented by the authors above are based on the presumption that a sufficient number of closed sales of comparable properties always exist in a finite time period such that a statistically reliable sample can be found. Excluding mass appraisal of residential properties where the sample sizes are large, the typical residential or commercial assignment always involves a search process to find recent closed sales that may exist.[1]

The first point missed is that the statistical tests and analysis used must be examined in a real-world environment that is characterized by statistical parameters that are *consistently unreliable because the sample size of closed sales, and in many cases the entire population, is always small.* Common statistical analysis that relies on a mean and variance is not applicable because the underlying assumptions that the error terms be normally distributed with zero mean, unit variance and zero covariances among the errors, are not usually satisfied. The perennial task of the analyst is to apply the

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appropriate statistical technique with a small sample size—frequently less than five—that will extract the correct market trend that the informed buyer is using to make a purchase decision.

The second point not discussed in any of these articles is the specific task of the analyst in the valuation assignment. This person knows in advance that the number of closed sales will be small, the property and financial characteristics most likely will be very heterogeneous, and as a result, the commonly taught statistical methods of analysis will not be applicable. Consequently, the analyst will search for the *properties that are the most comparable to the subject using a finite set of per-unit measures that reflects the buying trend*.[2] In practice, these best, recently sold, most comparable properties are the ones with the *fewest adjustments in the sales comparison approach*.[3] Any comparable sale that is so similar to the subject that the sales adjustment grid reveals very few minor adjustments or none at all must be very similar to the subject. This is the property that will be used with the largest emphasis in the final estimate of value from the sales comparison approach.

The selection of the "best comp" that illustrates the smallest number of adjustments is an application of the minimum variance suggested by Vandell or even a coefficient of variation recommended by Gau et al. "Best" implies minimum variance. The point here is that the mean and variance cannot be extracted accurately with a small sample from an unknown distribution. The theory is good, but not measurable or applicable.

The theme in this discussion is that the valuation business operates in this manner, and suggested statistical techniques for improving the selection of comparable properties must be evaluated in this environment. Presuming this is correct, the real-world scenario is that the typical valuation analyst might possess two-four recent closed sales that are possible candidates for the one that is the most comparable to the subject. The hypothesis advanced in this note is that the analyst *must rank* these candidates from the best to the worst. The best is defined as the sale that is the most comparable in that it exhibits the lowest frequency or smallest dollar amount of adjustment in the sales comparison approach, and the worst is the one with the highest frequency of adjustments or possibly the largest dollar amount of net adjustment.

Illustration

An income-producing property in a common assignment such as an apartment complex is valued by typical per unit measures selected from a group such as the following:

	Comparables			
	A	B	C	D
Sales price	$7.0m	$2.05m	$2.59m	$5.6m
Time of sale	1 yr	1 yr	1 mo	1 yr
Units	400	90	114	384
Rooms	1724	455	678	1536
Rooms/unit	4.31	5.06	5.94	4.00
PGI	1,070,650	305,900	435,625	800,000

Rent/room/no.	51.75	56.03	53.54	43.40
Price/room	4060	4505	3820	3646
Price/unit	17500	22780	22719	14585
GRM	6.54	6.70	5.95	7.00

Most likely, sale D will be eliminated as one of the main indicators of buying trend, rent per room, is much lower than the market trend of 56.03, 53.54 and 51.75 shown by the other three. Thus, a simple paired comparison eliminated D as it was ranked the lowest using a per-unit measure that is typical to this market.[4] A mean, standard deviation and minimum variance parameter with only four numbers is not meaningful. Further, a simple arithmetic average of any four numbers for a per-unit estimate does not allow for any judgment to be made on the quality of the data from each source or the task of the appraiser.

The above authors recommend the use of their statistical tools in the optimum selection of comparables when the appropriate question is the optimum selection of the *appropriate per-unit measures of comparison*. With a permanently small sample, the question to the appraiser is one of which estimate of value to use among the properties available.

In this example, the per unit measures of price-per-room, price-per-unit, and the gross rent multiplier (GRM) from property A were used by the analyst to estimate value. This comparable exhibited the highest number of elements of comparison that were similar to the subject. The three per-unit measures from property A were applied to the subject and produced estimates of value equal to $3,717,000, $3,710,000 and $3,712,000, respectively. A test of the other per-unit measures when applied to the subject produced numbers that appeared to be outliers relative to the range of the others, and were eliminated.

This situation of perpetually unreliable samples combined with a necessity to rank the properties from best to worst requires a statistical technique that incorporates both. Unfortunately, our literature and statistical texts do not contain a variety of tools that can be used with small sample sizes, and the ranking of data compounds the difficulty of identifying an appropriate choice.[5]

One purpose of this note is to suggest that rank transformation analysis can be used with small sample sizes and is more appropriate than minimum variance, the coefficient of variation, or ordinary least squares multivariate regression.

Extremely Small Samples

Given the typical appraisal situation of extremely small samples of recently sold comparable properties, say less than five, the need of the appraiser to select the "best" comparable, and the derived result that the sample of comps will be ranked from best to worst, the use of any known statistical tools is almost nonexistent. Econometricians have developed tools that work well only in macroeconomic settings that contain larger numbers.

This is a possible explanation for the continuation of paired-sale analysis that is commonly taught among appraisal organizations. It substitutes a procedure with some subjective decisionmaking for statistical analysis. A student is taught, first, to select the possible set of comparable properties using the nine-step process known as the "elements of comparison."[6] Second, the student extracts buying trends called "per-unit comparisons" using paired-sale analysis. Third, the per-unit measures are entered on a grid as adjustments that are used to locate the best comparable property.

Statistical Reliability. A small sample can contain data of comparable properties that provide unbiased and efficient statistical parameters because the underlying variance of the error terms satisfies the necessary (0,1) distribution. How does the analyst determine the variance *with a high level of reliability*? A small sample does not mean that a statistical model such as regression or minimum variance does not work. It means that the analyst does not know that the derived parameters are statistically reliable.

One argument for increasing the reliability is through continual sampling with replacement. This works well as the sample size is increased, but is rather pointless in a small sample when the data choices are severely limited.

Unanswered Question. The unanswered question is the accuracy of the valuation estimate using a minimum variance or coefficient of variation technique compared to the paired-sales procedure. Most likely, the paired-sale appraisal procedure will remain until more unbiased statistical tools are developed and shown to produce better accuracy in the estimate of value.

Ranking Techniques

Rank Transformation Regression

Rank transformation regression (RTR) has been shown in the statistical literature by Conover and Iman (1979, 1980, 1981) to be applicable in small sample sizes. Also, it reduces the influence of outliers, and generally produces a normal distribution of the error terms. RTR suffers from the same requirement imposed on other statistical techniques in that the sample size must be sufficiently large to produce meaningful estimates.

Their research shows that the rank transformation applies to all distributions. Using their terminology, let X_{ij} be the jth observation vector from population i, $j=1,2,\ldots n$ and $i=1,\ldots k$. The p-components of X_{ij} are denoted X_{ijm}, $m=1,2,\ldots p$. The RTR ranks the mth component of all observations, X_{ij}, from the smallest (rank 1) to the largest (rank $N=n_1\ldots+n_k$). Each component ($m=1$) to ($m=p$) is ranked separately. The value of each variable of a multivariate sample (cardinal data) is replaced by its rank from 1 to n (ordinal data) for all observations. Conventional regression is then performed on the ranks.

This technique is no better or worse than conventional multivariate analysis in eliminating autoregression and multicollinearity. The best combination of independent variables and the most robust model are found using commonly known methods. Also, it is not necessary to standardize or take logs of the raw data, as the monotonic transformation results in the same ranks (Iman and Conover, 1979, p. 500).

RTR has several additional advantages over other statistical methods, such as OLS and the coefficient of variation. First, the presence of outliers forces the analyst to edit the preliminary data, as the normality assumptions can be easily violated in OLS. It also drastically changes the coefficient of variation if the outliers remain in the data set. Typically, the analyst eliminates these observations using personal judgment on local market trends which makes the analysis highly subjective. RTR does not require the outliers to be removed.

Also, Hettmansperger and McKean (1978) show that rank procedures do not lose power or efficiency compared to results with cardinal data. Accuracy does not necessarily suffer because information is foregone when the cardinal data are eliminated in favor of the ordinal replacements.

Cronan, Epley and Perry (1986) compared the results of RTR with the most robust OLS analysis in an attempt to estimate the value of a single-family structure. The mean absolute deviation between the actual and the estimated price was consistently lower with the RTR method.

The RTR is an attempt to replicate the price that the buyer actually pays, which is the product of a number of integrated property characteristics that are not priced separately. The analyst's goal when using this procedure is to estimate the sales price accurately and underemphasize the interpretation of the coefficients. The signs and weights on the RTR variables do not carry the same interpretation as OLS.

Preference and Attribute Rankings

Perceived attribute ranked data can contain significant biases if the data are pooled and if the ranking alternatives are larger than four (Ben-Akiva, Morikawa and Shiroishi, 1992). The ranked data can be transformed into equivalent pairwise choices for use in a linear programming model (Garrido and Ortuzar, 1994). Also, ordinal data and ranks have been shown to classify more accurately the variables applied by Moody's when assigning BHC commercial paper ratings (Perry and Cronan, 1986).

Selection of Comparables from a Large Residential Databank

Recent efforts by Fannie Mae and The Mortgage Company to underemphasize the use of fee appraisers and develop massive databanks from closed residential sales makes this discussion of comparable selection more critical. The obvious intent is to estimate the value of the underlying loan collateral from statistical techniques that will extract total adjustments from the data set and apply them to a subject property.

The eventual accuracy and success will depend on two statistical and one appraisal question(s). The first is the ability of the statistical program to select the best comparables for the necessary adjustments. The second is the reapplication of the total adjustment to a specific subject. The third is the extent and purpose of the subject property inspection. All are topics for future research.

The inherent fallacy in valuation of property by a regression equation from a large databank is that the sales of past properties are used to value the existing stock of unsold properties without an inspection. As soon as an inspection is required, subjective adjustments must be allowed that introduce the complete appraisal process into the valuation. Valuation of the subject property using statistical analysis without an external inspection means that the lending establishment has projected a local risk-of-loss that is acceptable for errors made by reapplication of adjustments to the subject. Only time will determine if the saving in closing costs is less than the potential loss from foreclosure expenses.

This statistical effort has been feasible because Fannie Mae and The Mortgage Company are major investors in the secondary mortgage market and can require the appraisal (or some form of it) to be a part of the mortgage package. Most likely, this will not exist in the commercial sector as no one investor controls a significant proportion of the secondary market. Also, residential sales are more accessible to the users of this information than commercial transactions. The latter information may exist within a relatively closed set of professional users of the data who are secretive about releasing this information to a potential competitor. In sum, the large databanks for commercial properties will remain, in the short run, the domain of private analysts and professional users.

Conclusion

This note has reexamined the recommendations by Vandell, Gau et al. and Green for the use of the minimum variance and coefficient of variation criteria in the optimum selection of comparable properties. These authors have underemphasized the point that the appraiser operates in an environment where all sample sizes are small and all extracted per-unit measures from the comparable sales are consistently unreliable. Also, the appraiser ranks the available comparable sales from best to worst and uses the per-unit extractions primarily from the best comparable(s). A minimum variance or a coefficient of variation statistic will most likely be biased in this scenario.

The rank regression technique is appropriate as it does not depend upon the underlying distribution. It does not require the analyst to edit the preliminary data for outliers, and it can be used with a small sample.

The three-step market-extraction technique using paired-sale analysis has been substituted for common statistical parameters because the estimation tools are not reliable for small *n*. Most likely, this procedure will remain until small-sample econometric analysis produces estimates of value that are more accurate.

Notes

[1]The analyst should not confuse the models often employed in mass appraisal with the valuation procedure typically used in an appraisal assignment with one subject property. The differences have been recognized by the Appraisal Standards Committee in Standard 1 that covers the contents of a real property appraisal and a completely different standard, number 6, that covers mass appraisal. For example, 6-3, 6-4 and 6-5 discuss the calibration of models, the quantity and quality of factual data that are sufficient to produce a credible appraisal, goodness-of-fit, hold-out samples, and analysis of residuals. The contents in Standard 6 strongly imply a large sample size.

Further, mass appraisal techniques have been illustrated in the literature almost exclusively on residential properties only. The same statistical procedures have not successfully provided reliable estimates of value for income-producing properties due to restricted sample sizes and differences in definitions of variables between classes of properties.

[2]The selection of the single comparable property as the task of the appraiser in the proper application of the valuation process is offered here without empirical proof. The justification is found in the discussion of paired-sale analysis and the concept of reconciliation. For example, the Appraisal Institute (1992, p. 401) explains that "Comparable E is the property most similar to the subject and therefore may be accorded the greatest weight. On the basis of the indicated range of value and the weight placed on comparable E, a single point estimate indication of $74.00 per square foot of rentable area may be concluded."

Reconciliation involves a ranking of the selected comparables. The Appraisal Institute (1992, p. 406) explains this process in the sales comparison approach as "The market data grid can be used to rank sales in terms of their comparability to the subject property and to find which sales require the least total adjustment. The sales that occurred nearest to the appraisal data, those that require the least adjustments, and those that are the most similar to the subject are given the most weight in the sales comparison approach."

[3]The "best" comparable property can be the one with either the least absolute number of adjustments or the least amount of dollar adjustment. For example, a comparable with only one adjustment that is a small dollar amount is ideal. A subjective decision must be reached when comparable 1 has one adjustment with a large number of dollars, and comparable 2 has two adjustments with a smaller number of total dollars.

[4]See Appraisal Institute (1983, pp. 323–25) for data and a similar discussion of the selection of per-unit measures.

[5]The purpose of this study is to extend the discussion and search for statistical tools to be used with small samples. However, Celec (1982) provides a proof that the coefficient of variation can provide incorrect rankings in a situation where the underlying utility function is unknown and the decisionmaker prefers more to less. His illustration with a choice of two assets is very similar to the appraiser's choice between two alternative comparable properties. His conclusion is that this tool should not be used in comparing the relative desirability of competing assets.

[6]See Appraisal Institute (1992, Ch. 17).

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